The $N^{7/5}$ Law for Charged Bosons

Joseph G. Conlon$^1$, Elliott H. Lieb$^2$** and Horng-Tzer Yau$^2$***

$^1$ Department of Mathematics, University of Missouri, Columbia, MO 65211, USA
$^2$ Departments of Mathematics and Physics, Princeton University, P.O.B. 708, Princeton, NJ 08544, USA

Abstract. Non-relativistic bosons interacting with Coulomb forces are unstable, as Dyson showed 20 years ago, in the sense that the ground state energy satisfies $E_0 \leq -AN^{7/5}$. We prove that $7/5$ is the correct power by proving that $E_0 \geq -BN^{7/5}$. These bounds prove that the Bogoliubov type paired wave function correctly predicts the order of magnitude of the energy.

I. Introduction and Background

Twenty years ago Dyson and Lenard [5] proved the stability of ordinary non-relativistic matter with Coulomb forces, namely that the ground state energy, $E_0$, of an $N$-particle system satisfies $E_0 \geq -A_1 N$ for some universal constant $A_1$. In ordinary matter, the negative particles (electrons) are fermions. At the same time, Dyson [4] proved that bosonic matter is definitely not stable; if all the particles (positive as well as negative) are bosons then $E_0 \leq -CN^3/4$. Dyson and Lenard [5] did prove, however, that $E_0 \geq -C N^{3/4}$. This $3/4$ law is also validated here by showing that $E_0 \geq -D N^{1/4}$. These bounds prove that the Bogoliubov type paired wave function correctly predicts the order of magnitude of the energy.

In this paper we prove that the $N^{7/5}$ law is correct for bosons by obtaining a lower bound $E_0 \geq -A_4 N^{7/5}$. As is well known, the bosonic energy is the absolute lowest energy when no symmetry restriction is imposed.

It may appear that the difference between $5/3$ and $7/5$ is insignificant, especially since bosonic matter does not exist experimentally, but that impression would fail to take into account the essential difference between the ground states implied by