

Current Algebras in $d + 1$ -Dimensions and Determinant Bundles over Infinite-Dimensional Grassmannians[★]

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Abstract. We extend the methods of Pressley and Segal for constructing cocycle representations of the restricted general linear group in infinite-dimensions to the case of a larger linear group modeled by Schatten classes of rank $1 \leq p < \infty$. An essential ingredient is the generalization of the determinant line bundle over an infinite-dimensional Grassmannian to the case of an arbitrary Schatten rank, $p \geq 1$. The results are used to obtain highest weight representations of current algebras (with the operator Schwinger terms) in $d + 1$ -dimensions when the space dimension d is any odd number.

1. Introduction

In this paper we generalize some results of Pressley and Segal [PS] on the determinant line bundle over infinite-dimensional Grassmannians and on central extensions of infinite-dimensional linear groups. The ultimate aim is to obtain linear representations of current algebras arising in quantum field theory in $3 + 1$ -dimensions. In particular, we want to construct a generalization of the fermionic Fock representation of current algebras in $1 + 1$ -dimensions (including the Schwinger term), adapted to the $3 + 1$ -dimensional case. We have a partial resolution to this problem.

We are able to construct a highest weight representation for the $3 + 1$ -dimensional current algebra, including an explicit realization of the highest weight vector (= vacuum) as a section of the dual Det_2^* of the determinant bundle, Det_2 over a Grassmannian Gr_2 , which contains the Grassmannian Gr_1 studied in [PS]

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