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Geometric Realization of Conformal Field Theory on Riemann Surfaces

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Abstract. Conformal field theory on a family of Riemann surfaces is formulated. We derive equations of motion of vacua which are parametrized by moduli of Riemann surfaces and show that these vacua are characterized uniquely by these equations. Our theory has a deep connection with Sato's theory of KP equations.

0. Introduction

Recently it has been recognized that the conformal field theory (CFT) on Riemann surfaces of arbitrary genus plays an essential role to understand the profound mechanism of the string theory [F.S.; Fr.]. Among others very important insights have been brought by a formulation of the bosonization rule [D.J.K.M.; A-G.B.M.N.V.; E.O.; V.V.] and an observation that the Virasoro (energy-momentum tensor) operator deforms the moduli of Riemann surfaces [E.O.; B.M.S.].

One approach to the CFT on Riemann surfaces is based on the path-integral method initiated by Polyakov [P.]. This approach can be regarded as a geometric one which is recently developed into the algebro-geometric level [B.K.].

Another approach to the CFT is an algebraic one based on the representation theory of the Virasoro algebra, and was initiated by Belavin, Polyakov and Zamolodchikov [B.P.Z.]. This approach has an essential connection with solvable models of statistical mechanics and Kac-Moody Lie algebras.

One of the aims of this paper is to unify these two approaches by constructing a CFT on a family of Riemann surfaces in an operator formalism. Another aim is to establish a solid mathematical basis for a class of CFT on Riemann surfaces.

The main ingredient of our theory is M. Sato's theory of KP equations [Sa.; S.S.]. Originally his theory was developed to solve a problem of soliton equations, but here we show that his theory actually covers the CFT on Riemann surfaces.

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