Commun. Math. Phys. 116, 235–245 (1988)

Communications in Mathematical Physics

Dirichlet Forms and White Noise Analysis

T. Hida[1], J. Potthoff[2], and L. Streit[3]

[1] Department of Mathematics, Nagoya University
[2] Fachbereich Mathematik, Technische Universität Berlin, D-1000 Berlin 12
[3] BiBoS, Universität Bielefeld

Abstract. We use the white noise calculus as a framework for the introduction of Dirichlet forms in infinite dimensions. In particular energy forms associated with positive generalized white noise functionals are considered and we prove criteria for their closability. If the forms are closable, we show that their closures are Markovian (in the sense of Fukushima).

1. Introduction

In the past decade the theory of Dirichlet forms [5] has become an increasingly important link between probability theory, analysis, quantum theory and stochastic mechanics [1–6]. The infinite dimensional case is of particular interest for the development of infinite dimensional analysis and of quantum and stochastic models with infinitely many degrees of freedom [1, 14, 19, 20].

Here white noise analysis [7–9, 11, 13] turn out to offer a particularly suitable framework; the present article is just a first step towards the filling out of this frame.

In Sect. 2 we assemble some pertinent material from white noise analysis, in particular concerning positive white noise functionals and their representation by measures. As in the finite dimensional case the underlying nuclear rigging is far from unique, and different alternatives should be explored, with a view towards different applications.

In Sect. 3 we construct energy forms from positive generalized functionals of white noise. We give criteria for the admissibility of these functionals, so that the forms correspond to positive self-adjoint operators, generalizing the generator of the infinite dimensional Ornstein-Uhlenbeck process. We show that the construction goes beyond the case of measures which are absolutely continuous with respect to white noise; i.e. much wider classes of sample functions are allowed to occur. Finally we demonstrate the Markov property for the forms that we construct and conclude the paper by commenting on the Markov processes, which are generated by the Markovian semigroups associated with our forms.