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On the Large Order Behavior of Φ_4^4

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Abstract. We continue the rigorous study of the large order behavior of the perturbation series for the φ^4 model in 4 dimensions started in [1]. In this paper we prove a result announced in [1]. We show that the exact radius of convergence of the Borel transform of the renormalized perturbation series for φ_4^4 is greater than or equal to the expected value given by the position of the first "renormalon" [2]. This result holds for any vector $(\boldsymbol{\varphi}^2)^2$ model with N components, and makes use of the "Lipatov bound" of [1]. This result is based on a partial resummation of counterterms similar to the one of [3], but in a phase-space analysis of the renormalized series.

I. Introduction

A) The Renormalon Problem

The large order behavior of the renormalized series for φ_4^4 field theory is expected to be governed by the first "renormalon" singularity in the Borel plane [2]. It happens indeed that this singularity, which should exist only in dimension 4 where the theory is renormalizable, is closer to the origin than the "instanton" singularity which is responsible for the "Lipatov" large order behavior [4] of φ^4 series. Therefore this Lipatov behavior is only expected to hold for the lower dimensions 1, 2, and 3, where the φ^4 theory is superrenormalizable. Although this "Lipatov" behavior in the superrenormalizable domain has now been rigorously established, there is no theorem, up to now, on the existence of a single "renormalon" singularity, except in the trivial case of "infinite component" vector models.

A rigorous study of renormalons is interesting for several reasons. The renormalon is the modern version of the "Landau ghost" [14]. As shown in [2] by Parisi and 't Hooft, for those renormalizable theories which are not asymptotically free (i.e. the one loop β function coefficient is positive), one expects singularities in

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