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Invariants for Smooth Conjugacy of Hyperbolic Dynamical Systems. IV

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Abstract. We show that if two C^∞ transitive Anosov flows in a three-dimensional manifold are topologically conjugate and the Lyapunov exponents on corresponding periodic orbits agree, then the conjugating homeomorphism is C^∞.

I. Introduction and Statement of Results

The purpose of this paper is to present a unified approach to the problem of smooth conjugacy of Anosov systems (both flows and diffeomorphisms) in low dimensional manifolds. Our results extend those of [Ll] for diffeomorphisms in that no proximity assumption is made. Also, the results for flows, [MM], are extended to cover arbitrary flows instead of one-parameter families of them. We also recover the result of [MM] that all Anosov diffeomorphisms of a two-dimensional manifold with constant Lyapunov exponents in periodic orbits are C^∞ conjugate to linear automorphisms. Finally, we show how the important result of Feldman and Ornstein, [FO], on C^1 conjugacy of geodesic flows implies C^∞ conjugacy.

We have known from Palis that Ghys has suggested a different approach using one-dimensional expansive maps to prove C^1 regularity of the conjugacy.

Our results are the following:

Theorem 1. *Let X, Y be two C^∞ transitive Anosov vector fields in a compact three-dimensional manifold. If they are C^0 conjugated and the Lyapunov exponents at corresponding periodic orbits are the same, then the conjugating homeomorphism is C^∞.*

Remark. Notice that the hypothesis about the Lyapunov exponents follows from C^1, or even Lipschitz, conjugacy. Hence, C^1 conjugacies are C^∞ for the systems considered in Theorem 1. This remark also applies to Theorem 2.

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