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On a Generalised Fourier Transform of Instantons Over Flat Tori

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Abstract. Recently P. Braam pointed out that Nahm's adaption of the ADHM procedure to the case of monopoles equally well applies to instantons over flat tori, relating them to instantons over the first Brillouin zone. We show that this construction has an inverse. Hence the Nahm transform actually is a duality transform.

Introduction

It is an enticing thought that Bloch's analysis [1] might be the appropriate means for studying Yang-Mills fields over flat tori. Due to the non-linear character of the Yang-Mills equations, however, one may doubt that such a method is expedient. In this light it is a remarkable observation of Braam [2] that Nahm's considerations on the construction of monopoles [3] may be translated to instantons over flat tori. While Braam associates an instanton over the respective Brillouin zone $\hat{T}$ (dual torus) to every instanton over a torus T, we shall give a formulation of his observation which will allow us to prove the invertibility of this construction: after repeated application of the *Nahm transform* we recover the original instanton.

The ideas presented here may also be interpreted as an explicit version of some ideas of Mukai [4] if one takes into account the theorem of Donaldson, Yau, and Uhlenbeck [5] relating the stability of vector bundles to the existence of Hermitian-Einstein connections.

0. Vector Bundles over Flat Tori

We consider a four dimensional flat torus $T=\mathbb{R}^4/\Lambda$, defined by a lattice $\Lambda\subset\mathbb{R}^4$. Every vector bundle over T may be described by matrix-multipliers. A matrix-multiplier e of rank n is defined to be a map $e:\mathbb{R}^4\times\Lambda\to GL(n,\mathbb{C})$, $e:(x,\lambda)\mapsto e(x,\lambda)$, satisfying the condition

$$e(x,\lambda+\mu)=e(x+\mu,\lambda)e(x,\mu)\,. \tag{0.1}$$