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A Power Counting Theorem for Feynman Integrals on the Lattice

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Abstract. A convergence theorem is proved, which states sufficient conditions for the existence of the continuum limit for a wide class of Feynman integrals on a space-time lattice. A new kind of a UV-divergence degree is introcduced, which allows the formulation of the theorem in terms of power counting conditions.

1. Introduction

Feynman integrals on a cubic, four-dimensional lattice have a very specific structure. In momentum space the integration domain is the Brillouin zone (BZ), hence compact for every non-vanishing lattice spacing a. Instead of being rational the integrand is a periodic function. If none of the propagators has vanishing mass, and so we will assume throughout this paper, a Feynman integral is absolutely convergent for every finite lattice spacing. We want to discuss the behaviour of such inegrals if the cutoff is removed, i.e., if the lattice spacing a tends to zero.

There exists the well known power counting theorem of Hahn and Zimmermann [1] which states sufficient conditions for the absolute convergence of ordinary Feynman integrals. Convergence depends on the behavior of the integrand in various sections of the integration domain where some or all integration momenta get large. This behavior is described by use of UV-divergence degrees of the integrand with respect to so-called Zimmermann subspaces, i.e., special classes of affine subspaces of the integration momenta. If the divergence degrees with respect to all these subspaces are smaller than zero, the Feynman integral will be absolutely convergent. Unfortunately, this power counting theorem assumes a rational structure of the integrand and hence does not apply to diagrams with a lattice cutoff. Similar problems occur in connection with Weinberg's power counting theorem [2]. In fact, it is meaningless to discuss naively large momenta on the lattice, the integrand of a Feynman integral being periodic. Actually, if convergence holds,

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