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On the Borel Summability of Planar Perturbation Series: Transition to Minkowski Space

Christoph Kopper*,**

Institute for Theoretical Physics, Princetonplein 6, P.O. Box 80.006, NL-3508 TA Utrecht, The Netherlands

Abstract. In recent years 't Hooft and Rivasseau proved the Borel summability of planar asymptotically free massive theories in Euclidean space. The corresponding Borel sums in Minkowski space are shown to exist as linear functionals if the Euclidean counterparts are bounded polynomially in momentum space and fulfill certain analyticity conditions. Both can be verified in massive planar "wrong sign" φ_4^4 using Rivasseau's approach. The functionals alternatively are densely defined and unbounded on an L^p space or bounded on (the whole of) a Banach space with a more restrictive norm.

I. Introduction

A few years ago 't Hooft [1, 2] and Rivasseau [3] established the Borel summability of planar asymptotically free theories without any massless particles in four-dimensional Euclidean space. We shall try to translate those results to Minkowski space.

Our work is based on the methods of Rivasseau which explicitly display the relation between the renormalized perturbative series for a given one particle irreducible (1PI) Green function and its Borel sum which is shown to be identical to a sum over certain dressed Feynman amplitudes. He restricts to a $U(N)$ [or $O(N)$] symmetric massive scalar matrix field theory with the sign of the scalar self-coupling chosen such that the theory becomes asymptotically free. (The case where the scalar fields are in the fundamental N-vector representation is far simpler. The limit $N \to \infty$ can even be obtained analytically, see e.g. [4].) In principle this method should be applicable to more general examples as regarded by 't Hooft [1, 2], i.e. planar gauge theories where all particles become massive through the Higgs mechanism and all couplings asymptotically free on imposing suitable

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** After October 1, 1987: Max-Planck-Institut für Physik, Föhringer Ring 6, D-8000 München 40, Federal Republic of Germany