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Analyticity Properties of the Surface Free Energy of the Ising Model

C.-E. Pfister[1] and O. Penrose[2]

[1] Département de Mathématiques, EPFL-Lausanne, CH-1015 Lausanne, Switzerland
[2] Department of Mathematics, Heriot-Watt University, Riccarton, Edinburgh, EH14 4AS, UK

Abstract. For an Ising ferromagnet with nearest-neighbour interactions of strength K and surface magnetic field h, the surface free energy in the presence of a positively (or negatively) magnetized zero-field bulk phase is shown to be analytic in h for $|\mathrm{Re}\, h| < K - \alpha/\beta$, where $\alpha = 2.96\ldots$ and β is the inverse temperature. This puts the lower bound $K - \alpha/\beta$ on the values of h at which wetting and layering transitions can take place.

1. Introduction

Surface phase transitions such as the wetting transition can be modelled by an Ising ferromagnet in which the magnetic field applied on a part of the boundary of the lattice is different from that in the bulk. (This is equivalent to a binary mixture on a lattice in contact with a wall which preferentially adsorbs one or the other of the two components, depending on the sign of the surface magnetic field h.) The surface and bulk magnetic fields, which may be in opposite directions, compete to determine the state near the surface. At temperatures T below the bulk critical temperature T_c this competition persists even in the limit of zero bulk field: if the part of the lattice far from the surface is in the negatively magnetized phase, corresponding to an infinitesimally small negative bulk field, then it is still necessary for the surface field h to exceed a certain positive value $h_w(T)$ if the surface layer of positively magnetized phase near the surface is to invade the bulk. This "wetting" transition at $h = h_w$ was rigorously established by Abraham [1] for the two-dimensional case, and by Fröhlich and Pfister [2] for three or more dimensions. A summary of the known rigorous results for such models is given in [3].

One way to study possible phase transitions in the surface is to look at the surface free energy, whose precise definition is given later in this paper. It is important to realize that the surface free energy is a single-valued function of the parameters of the model (surface magnetic field, bulk magnetic field and temperature) *only* if there is a unique bulk phase. Therefore, with the bulk field held