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On the Linearized Relativistic Boltzmann Equation

I. Existence of Solutions

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Abstract. The linearized relativistic Boltzmann equation in $L^2(\mathbf{r},\mathbf{p})$ is investigated. The detailed analysis of the collision operator L is carried out for a wide class of scattering cross sections. L is proved to have a form of the multiplication operator $\nu(\mathbf{p})$ plus the compact in $L^2(\mathbf{p})$ perturbation K. The collisional frequency $\nu(\mathbf{p})$ is analysed to discriminate between relativistic soft and hard interactions. Finally, the existence and uniqueness of the solution to the linearized relativistic Boltzmann equation is proved.

1. Introduction

In the standard approach to the relativistic kinetic theory one assumes the Boltzmann equation as an evolution equation for the one-particle distribution function [1, 2]. One of the main mathematical problems one faces in that approach is to prove, under physically reasonable assumptions, existence of a unique solution of the Cauchy problem. Such a proof depends crucially on a specific choice of a function space one uses to describe a physical system. Several interesting results concerning solutions of the Boltzmann equation in general relativity have already been obtained [3–5]. For functions of a compact support bounded by $\exp[-\beta_\alpha(x)p^\alpha]$ Bichteler [3] proved local existence of a solution to the Boltzmann equation under the assumption that the total scattering cross section is finite. A similar result, but in Sobolev spaces and with additional assumptions on a form of cross section, was obtained by Bancel [4]. The Sobolev spaces are also appropriate for analysis of the coupled Boltzmann and Einstein-Maxwell equations. The Cauchy problem for such a system of equations has been solved by Bancel and Choquet-Bruhat [5].

It is still not known whether solutions in those spaces allow for any hydrodynamical approximation. On the other hand, the detailed analysis of the nonrelativistic Boltzmann equation, including rigorous justification of the hydrodynamic approximation, has been given by Grad [6, 7], Ellis and Pinsky [8], Nishida [9], and Kawashima et al. [10] for a different space of functions correspond-