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An n-Dimensional Borg–Levinson Theorem

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Abstract. We show that the potential q is uniquely determined by the spectrum, and boundary values of the normal derivatives of the eigenfunctions of the Schrödinger operator $-\Delta + q$ with Dirichlet boundary conditions on a bounded domain Ω in $\mathbb{R}^n$. This and related results can be viewed as a direct generalization of the theorem in the title, which states that the spectrum and the norming constants determine the potential in the one dimensional case.

1. Introduction

Let $q(x)$ be a real-valued potential in $L^\infty[0,1]$ and let $y(x,\mu)$ solve the initial value problem

$$\begin{aligned} -y'' + qy &= \mu y \quad \text{for} \quad x \in (0,1), \\ y(0,\mu) &= 0, \\ y'(0,\mu) &= 1. \end{aligned}$$

Define the sequence $\{\mu_i(q)\}_{i=1}^{\infty}$ of Dirichlet eigenvalues by the condition

$$y(1,\mu_i) = 0$$

and define the norming constants c_i by

$$c_i(q) = \int_0^1 y^2(x,\mu_i)\,dx.$$

A well known result of Borg [B] and Levinson [L] is

Theorem 1.1. *Suppose that $q_1, q_2, \in L^\infty(0,1)$, are real-valued and that, for all i*

$$\mu_i(q_1) = \mu_i(q_2)$$

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