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Markov and Stability Properties of Equilibrium States for Nearest-Neighbor Interactions

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Abstract. Consider models on the lattice $\mathbb{Z}^d$ with finite spin space per lattice point and nearest-neighbor interaction. Under the condition that the transfer matrix is invertible we use a transfer-matrix formalism to show that each Gibbs state is determined by its restriction to any pair of adjacent (hyper)planes. Thus we prove that (also in multiphase regions) translationally invariant states have a global Markov property. The transfer-matrix formalism permits us to view the correlation functions of a classical d-dimensional system as obtained by a linear functional on a noncommutative (quantum) system in $(d-1)$-dimensions. More precisely, for reflection positive classical states and an invertible transfer matrix the linear functional is a state. For such states there is a decomposition theory available implying statements on the ergodic decompositions of the classical state in d dimensions. In this way we show stability properties of $\mathbb{Z}^d_{\mathrm{ev}}$-ergodic states and the absence of certain types of breaking of translational invariance.

0. Introduction

In this paper we study properties of and relations between the equilibrium states of some models with nearest-neighbor interactions on the lattice $\mathbb{Z}^d$. As a main tool we use a transfer-matrix formalism. This formalism enables us to prove a global Markov property with respect to (hyper)planes for invariant Gibbs states. This property entails that for such states the spins in $\{x\in\mathbb{Z}^d | x_1 > 0\}$ behave independently of the spins in $\{x\in\mathbb{Z}^d | x_1 < 0\}$ upon fixing the spins at the boundary $\{x\in\mathbb{Z}^d | x_1 = 0\}$.

Intuitively Markov properties and transfer-matrix techniques are closely related. The relation can be made explicit when considering a part of $\mathbb{Z}^d$, for instance $\bar{\Lambda}^{(N)} \equiv \{x\in\mathbb{Z}^d | |x_i| \leqq N \text{ for } i = 1, \ldots, d-1\}$ with $N\in\mathbb{N}$, which is infinite only in one direction. In this case one is effectively considering a one-dimensional system. On all of $\mathbb{Z}^d$ however, the relation between the existence of finite-volume transfer matrices and Markov properties is not immediate, although if one assumes Markov properties the introduction of an infinite-volume transfer operator is not difficult,