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Nijenhuis *G*-Manifolds and Lenard Bicomplexes: A New Approach to KP Systems*

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Abstract. We suggest a method to extend the theory of recursion operators to integrable Hamiltonian systems in two-space dimensions, like KP systems. The approach aims to stress the conceptual unity of the theories in one and two space dimensions. A sound explanation of the appearance of bilocal operators is also given.

1. Introduction

This paper deals with the theory of recursion operators for nonlinear Hamiltonian equations in two space dimensions. According to a common opinion [1, 2], the use of these operators cannot be extended beyond the theories in one space dimension. In fact, a new phenomenology occurs in two space dimensions, which seems to be incompatible with a classical recursion scheme, its more impressive feature being the appearance of bilocal operators [1]. Our aim is to correct this opinion. We believe that it is due to an incomplete understanding of the potentialities of the method of recursion operators. In our opinion, a basic element of the theory has been missed, that is the role of the symmetry algebra. Usually, in fact, the recursion operators are coupled with a peculiar algebra of vector fields leaving them invariant. This algebra is (in some sense) trivial in one space dimension, and this fact explains why its role has been so far underestimated. It becomes crucial in two space dimensions. In the present paper the symmetry algebra is taken, since the beginning, as a fundamental element of the theory, at the same level of the recursion operator. This leads us to develop, in Sect. 2, the theory of the *Nijenhuis G-manifolds*. (For conceptual reasons, we prefer to use the name of Nijenhuis tensor instead of the more common, but also less specific, term of recursion operator.) The main outcome is the quite natural notion of *Lenard bicomplex*. It is the basic tool allowing us to deal in a surprisingly unified way with the theories both in one and two space dimensions. The same differences which seemed before to cleanly mark the two cases, appear now of

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