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The Beltrami Spectrum for Incompressible Fluid Flows

Peter Constantin[1,*] and Andrew Majda[2,**]

[1] Department of Mathematics, The University of Chicago, Chicago, IL 60637, USA
[2] Department of Mathematics, and Program in Applied and Computational Mathematics, Princeton University, Princeton, NJ 08544, USA

Abstract. Recently V. Yakhot, S. Orszag, and their co-workers have suggested that turbulent flows in various regions of space organize into a coherent hierarchy of weakly interacting superimposed approximate Beltrami flows. A mathematical framework is developed here to study organized Beltrami hierarchies in a systematic fashion. This framework is applied to several important classes of examples with universal Beltrami hierarchies. An analysis of the persistence of such Beltrami hierarchies is also presented for general solutions of the Navier-Stokes equations.

Introduction

Recently V. Yakhot, S. Orszag, and their co-workers [3, 4] have suggested that in various regions of space, turbulent flows organize into a coherent hierarchy of weakly interacting superimposed approximate Beltrami flows. Their evidence for such behavior is based on detailed numerical experiments for channel flows and decaying homogeneous turbulence utilizing spectral methods; however the mechanisms for the existence of such a hierarchy are not understood.

In this paper we develop a mathematical framework to study organized Beltrami hierarchies and then we analyze this structure in solutions of the Navier-Stokes equations. We advocate the theoretical framework presented in detail in Sect. 1 as a readily implemented new diagnostic for further numerical tests with spectral codes for the existence of weakly interacting Beltrami hierarchies. In Sect. 1 we also describe the concept of *Beltrami spectrum* as an effective measure of the extent to which a given incompressible velocity field is an organized superposition of weakly interacting Beltrami flows. Expressions yielding the same numerical value as the Beltrami spectrum were mentioned in works on helical

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