Commun. Math. Phys. 115, 375–419 (1988)

Communications in
Mathematical
Physics

Recursion Operators and Bi-Hamiltonian Structures in Multidimensions. I

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Abstract. The algebraic properties of exactly solvable evolution equations in one spatial and one temporal dimensions have been well studied. In particular, the factorization of certain operators, called recursion operators, establishes the bi-Hamiltonian nature of all these equations. Recently, we have presented the recursion operator and the bi-Hamiltonian formulation of the Kadomtsev-Petviashvili equation, a two spatial dimensional analogue of the Korteweg-deVries equation. Here we present the general theory associated with recursion operators for bi-Hamiltonian equations in two spatial and one temporal dimensions. As an application we show that general classes of equations, which include the Kadomtsev-Petviashvili and the Davey-Stewartson equations, possess infinitely many commuting symmetries and infinitely many constants of motion in involution under two distinct Poisson brackets. Furthermore, we show that the relevant recursion operators naturally follow from the underlying isospectral eigenvalue problems.

1. Introduction

In recent years a deep connection has been discovered [1, 2] between certain nonlinear evolution equations in $1+1$, i.e. in one spatial and one temporal dimensions, and certain linear isospectral eigenvalue (or scattering) equations. These isospectral problems play a central role in developing methods for solving several types of initial value problems of the associated nonlinear evolution equations. The most well known such method, the celebrated inverse scattering transform (IST) method, deals with initial data decaying at infinity. However, the isospectral problem is also crucial for characterizing periodic [3] as well as self similar solutions [4].

It is quite satisfying, from a unified point of view, that the isospectral problems are also central in investigating the "algebraic" properties of the associated

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