Commun. Math. Phys. 115, 353–374 (1988)

Communications in Mathematical Physics

The Periodic Orbits of an Area Preserving Twist Map

S. B. Angenent*

Department of Mathematics, University of Leiden, Niels Bohrweg 1, Leiden, The Netherlands

Abstract. We study the oscillation properties of periodic orbits of an area preserving twist map. The results are inspired by the similarity between the gradient flow of the associated action-function, and a scalar parabolic PDE in one space dimension. The Conley-Zehnder Morse theory is used to construct orbits with prescribed oscillatory behavior.

1. Introduction

We shall consider a C^1 area preserving diffeomorphism $\tilde{F}$ of the cylinder $S^1 \times R$ onto itself. Such a diffeomorphism can be described by a mapping $F: R^2 \to R^2$ (its lift) given by $F(x, y) = (f(x, y), g(x, y))$, where x is the angle coordinate. The components of F satisfy the periodicity conditions

$$f(x+1, y) = f(x, y) + 1, \qquad g(x+1, y) = g(x, y).$$

The map F is said to be a *twist diffeomorphism* if $f(x, y)$ is an increasing function of y, and in fact

$$\partial_2 f(x, y) > 0 \tag{1.1}$$

holds for all (x, y) in R^2. Here ∂_k denotes differentiation with respect to the k-th argument.

We shall consider twist diffeomorphisms which satisfy the *infinite twist condition*, i.e.

$$\lim_{y \to \pm\infty} f(x, y) = \pm\infty$$

for any $x \in \mathbb{R}$. We shall study the set of periodic orbits of $\tilde{F}$.

The main feature which distinguishes twist maps from other area preserving maps is that they have a single valued generating function, i.e. there is a C^2 function

* Current address: Department of Mathematics, University of Wisconsin, Van Vleck Hall, 480 Lincoln Drive, Madison, WI 53706, USA