Commun. Math. Phys. 115, 301–351 (1988)

Communications in Mathematical Physics

Analytic Torsion and Holomorphic Determinant Bundles

III. Quillen Metrics on Holomorphic Determinants

Jean-Michel Bismut[1], Henri Gillet[2,*], and Christophe Soulé[3]

[1] Département de Mathématique, Université Paris-Sud, F-91405 Orsay Cedex, France
[2] Department of Mathematics, University of Illinois at Chicago, Chicago, IL 60638, USA
[3] CNRS LA 212 and IHES, 35, Route de Chartres, F-91440 Bures/Yvette, France

Abstract. In this paper, we prove that in the case of holomorphic locally Kähler fibrations, the analytic and algebraic geometry constructions of determinant bundles for direct images coincide. We calculate the curvature of the holomorphic Hermitian connection for the Quillen metric on the determinant bundle. We study the behavior of the Quillen metric under change of metrics in the fibers, and also on the twisting vector bundles. We thus generalize the conformal anomaly formula to Kähler manifolds of arbitrary dimension. We also study the Quillen metrics on determinants associated with exact sequences of vector bundles. We prove that the Quillen metric is smooth on the Grothendieck-Knudsen-Mumford determinant for arbitrary holomorphic fibrations.

Contents

1. The Analysis of Holomorphic Determinant Bundles 303

a) Assumptions and Notations 304
b) Description of the Determinant Bundle 304
c) A Holomorphic Structure on λ 305
d) A Metric and a Holomorphic Connection on λ 308
e) Evaluation of the Curvature of ${}^1V$ 309
f) Identification of the Metrics and Connections on λ and λ' 313
g) The Holomorphic Structure Does not Depend on the Kähler Metric in the Fibers . 314
h) Dependence of the Metric of λ on (g^Z, h^ξ) 318
i) The Curvature of λ for the Quillen Metric: The General Case 327

2. $\lambda = \lambda^{KM}$: An Analytic Proof. 328

a) Infinite Determinants and Exact Sequences. 329
b) Evaluation of $\| T(\bar{\partial}+v)\|$. 331
c) Multiplicativity Properties of $T(\bar{\partial}+v)$ 332
d) $T(\bar{\partial}+v)$ is the Knudsen-Mumford Section 334
e) The Case where π is Projective: λ is the Knudsen-Mumford Determinant. 336
f) A First Proof of Theorem 0.1. 338

* Supported by NSF Grant DMS 850248 and by the Sloan Foundation