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One-Dimensional Schrödinger Operators with Random Decaying Potentials

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Abstract. We investigate the spectrum of the following random Schrödinger operators:

$$H(\omega) = -\frac{d^2}{dt^2} + a(t)F(X_t(\omega)),$$

where $F(X_t(\omega))$ is a Markovian potential studied by the Russian school [8]. We completely describe the transition of the spectrum from pure point type to absolutely continuous type as the decreasing order of $a(t)$ grows. This is an extension to a continuous case of the result due to Delyon-Simon-Souillard [6], who deal with the lattice case.

1. Introduction

In this paper, we will study the one-dimensional Schrödinger operator:

$$H(\omega) = -\frac{d^2}{dt^2} + a(t)F(X_t(\omega)) \tag{1.1}$$

on $L^2(\mathbf{R}, dt)$, where $\{X_t(\omega); t \in \mathbf{R}\}$ is a Brownian motion on a compact Riemannian manifold M with the normalized Riemannian volume element μ as its marginal distribution. Then $\{X_t(\omega); t \in \mathbf{R}\}$ becomes a stationary ergodic process on M. We assume that $F \in C^\infty(M)$, $a \in C^\infty(\mathbf{R})$, $a(t)$ is non-increasing on $\mathbf{R}_+ = (0, \infty)$, $a(t) = a(-t)$, and $a(t) \to 0$ as $|t| \to \infty$. It is known that $H(\omega)$ defines a self-adjoint operator on $L^2(\mathbf{R}, dt)$.

For a self-adjoint operator H on a Hilbert space, we denote by ΣH, $\Sigma_{\mathrm{p}} H$, $\Sigma_{\mathrm{sc}} H$, and $\Sigma_{\mathrm{ac}} H$ spectrum, point spectrum, singular continuous spectrum and absolutely continuous spectrum of H respectively (see Kato [12]). Our interest here is to investigate the existence or non-existence of these components of the spectrum of $H(\omega)$. Since $a(t) \to 0$ as $|t| \to \infty$, $H(\omega)$ has only discrete spectrum on $(-\infty, 0)$ if any and $\Sigma H(\omega) \cap [0, \infty) = [0, \infty)$ (Reed and Simon [17]).