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Calabi-Yau Hypersurfaces in Products of Semi-Ample Surfaces

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Abstract. We study Calabi-Yau manifolds that are embedded as hypersurfaces in products of semi-ample complex surfaces. We classify the deformation classes of the latter and thereby achieve a classification of the Calabi-Yau manifolds that are constructed in this way. Complementing the results in the existing literature, we obtain the complete Hodge diamond for all Calabi-Yau hypersurfaces in products of semi-ample surfaces.

1. Introduction

In order to explore the phenomenological implications of the Superstring theories [1], originally defined in $9+1$ space-time dimensions, it was shown to be consistent to assume that the $9+1$-dimensional space-time locally has the form of $M_4 \times \mathscr{M}_{\mathrm{CY}}$, where M_4 is $3+1$-dimensional Minkowski space and $\mathscr{M}_{\mathrm{CY}}$ is a Calabi-Yau manifold of 3 complex dimensions [2, 3]. Calabi-Yau manifolds are compact and admit a Ricci-flat Kähler metric, i.e. have a vanishing first Chern class [4].

Several examples of such manifolds were analyzed in [2, 5, 6]. Upon restriction to massless states and compactification on a $\mathscr{M}_{\mathrm{CY}}$, the effective models exhibit $N=1$ supergravity and Yang-Mills interactions with the gauge group being a subgroup of $E_6 \times E_8$, coupled to matter superfields the spectrum of which is counted by topological invariants of $\mathscr{M}_{\mathrm{CY}}$. In particular, superfields that transform as $(\mathbf{27}, \mathbf{1})$ of $E_6 \times E_8$ are counted by the Hodge number $b_{1,2}$ while the $(\mathbf{27}^*, \mathbf{1})$-transforming superfields are $b_{1,1}$-fold degenerate.

In [7, 8] a huge family of Calabi-Yau manifolds was established all of which are embedded in products of complex-projective spaces as complete intersections of hypersurfaces (we shall adopt the CICY acronym [9] in what follows). Because of the fact that the Euler characteristic $\chi_E = 2(b_{1,1} - b_{1,2})$ for all Calabi-Yau manifolds, and since χ_E is computed straightforwardly for every case, the difference of the number of $\mathbf{27}$'s and $\mathbf{27}^*$'s of E_6 is readily obtained for models based on any of these manifolds. In contrast, the computation of $b_{1,2}$ and $b_{1,1}$ separately appears to be a much harder task.