BRST Cohomology in Classical Mechanics

Marc Henneaux[1,2,*] and Claudio Teitelboim[2,3]

[1] Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium
[2] Centro de Estudios Científicos de Santiago, Casilla 16443, Santiago 9, Chile
[3] Center for Relativity, The University of Texas at Austin, Austin, TX 78712, USA

Abstract. The classical (non-quantum) cohomology of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry in phase space is defined and worked out. No group action for the gauge transformations is assumed. The results cover, therefore, the general case of an "open algebra" and are valid "off-shell." Each cohomology class contains all BRST invariant functions with fixed ghost number (an integer) which differ from each other by a BRST variation. If the ghost number is negative there is only the trivial class whose elements are equivalent to zero. If the ghost number is positive or zero there is a bijective correspondence between the BRST classes and those of the exterior derivative along the gauge orbits. These gauge orbits lie in the phase space surface on which the gauge generators are constrained to vanish. The BRST invariant functions of ghost number p are then related to closed p-forms along the orbits. The addition of a BRST variation corresponds to the addition of an exact form. Some comments about the quantum case are included. The physical meaning of the classes with ghost number greater than zero is not discussed.

I. Introduction

The concept of ghost has steadily emerged as one of central importance in the study of the dynamics of physical systems endowed with a local geometric structure or, as one says, of gauge systems.

Ghosts were first encountered in quantum field theory as fields with the "wrong" relation between spin and statistics which were necessary in addition to those which appeared in the correspondent classical problem [1]. The ghost fields ensured that the theory would be unitary. This they did through contributions to virtual processes only, thus avoiding conflict with the spin-statistics theorem.

After the advent of the Faddeev-Popov measure for summing over classes of equivalence of gauge field histories [2], the ghosts were regarded as just an artifact leading to a useful representation of that measure. However they were restored to a somewhat more respectable role when it was realized that in more complicated

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