Commun. Math. Phys. 115, 191–212 (1988)

Communications in
Mathematical
Physics

String Structures and the Index of the Dirac–Ramond Operator on Orbifolds*

K. Pilch** and N. P. Warner***

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. We discuss the relationship between "string structures" and the topological class $\lambda \in H^4(M, \mathbf{Z})$ on non-simply connected manifolds. We also investigate to what extent the index formula for the Dirac–Ramond operator detects the class, λ.

1. Introduction

It is now evident that the Dirac–Ramond operator [1] plays an important role in physics and mathematics. In physics it was shown that the anomaly generating function in string theory [2, 3] could be derived as the character valued index of this operator [4, 5]. In mathematics a similar construction provided an explicit realization of the elliptic cohomology and was introduced to prove certain vanishing theorems [6–9] conjectured by Witten [10]. In fact by studying automorphisms of the Dirac–Ramond operator these can be now proven more directly [5, 11, 12].

This interplay between physics and mathematics resulted in a rather unusual chronological order for the discovery of the properties of the Dirac–Ramond operator. For example, the index formula was known even before the operator itself had been rigorously defined[1], and in particular, before the generalization of spin-structure had been properly understood.

In order to define a Dirac operator on a manifold, M, and hence discuss its index, the manifold must have a spin-structure. This is a topological restriction which is equivalent to requiring that the second Stiefel–Whitney class $w_2(M) \in H^2(M, \mathbf{Z}_2)$ vanishes. Physically, one can think of this like a Dirac quantization condition in the presence of the Dirac monopoles. On certain

* Supported by the NSF Grant #84-07109 and by the U.S. Department of Energy (D.O.E.) under contract #DE-AC02-76ER03069

** On leave from the University of Wroclaw, Wroclaw, Poland

***Present address: Theory Division, CERN, Geneva

[1] In fact, there is still no precise definition of the general Dirac–Ramond operator. The operator used in [12] is most probably equivalent to the high temperature limit of the operator on the loop space