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The Global Markov Property for Equilibrium States Which are Determined by Correlations in a Strip

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Abstract. Consider an interaction-round-face potential on the lattice $\mathbb{Z}^d$, which may include nearest-neighbor and next-nearest-neighbor pair interactions, as well as more general plaquette terms. Assuming some periodicity of the potential it is shown that, under the condition that equilibrium states can be distinguished by expectation values of sufficiently local observables, equilibrium states possess a global Markov property. The condition under which this Markov property is shown to hold is met in particular in case the equilibrium state is unique or determined by the magnetization. The proof is based on an application of the variational principle to states which are constructed like Markov chains.

1. Introduction

We consider systems of classical spins on $\mathbb{Z}^d$ with, per lattice point, values in a compact metric space. The Markov property entails that upon fixing the configuration on the boundary of a volume the distribution of the spin configurations inside the boundary becomes independent of the spin configuration outside the boundary. The very definition of Gibbs states for a finite range potential states that Markov properties hold for finite volumes, see e.g. [1]. In this case one speaks of a local Markov property. In case such a property holds for an infinite volume this is called a global Markov property.

Gibbs states for finite-range potentials possess thus local Markov properties. However, the question whether or not Gibbs states possess global Markov properties is not easy to answer. Indeed, for the three-dimensional Ising model Goldstein has constructed an example of a Gibbs state that does not have the global Markov property with respect to a plane, [1]. Other examples of this type are given in [10].

Global Markov properties are an important ingredient in Nelson's scheme of Euclidean field theory because these properties are essential in constructing the Hamiltonian of the theory (see e.g. [2]). Despite their importance, global Markov properties are known to hold in relatively few cases. These cases can be divided into two main groups: