

# Analytic Torsion and Holomorphic Determinant Bundles

## II. Direct Images and Bott-Chern Forms

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**Abstract.** In this paper, we derive the main properties of Kähler fibrations. We introduce the associated Levi-Civita superconnection to construct analytic torsion forms for holomorphic direct images. These forms generalize in any degree the analytic torsion of Ray and Singer. In the case of acyclic complexes of holomorphic Hermitian vector bundles, such forms are calculated by means of Bott-Chern classes.

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### Introduction

This is the second of a series of three papers devoted to the study of holomorphic determinant bundles and direct images. Parts I and III of this work will be referred

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