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Block Spin Approach to the Singularity Properties of the Continued Fractions

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Abstract. The massless singularity of a ferromagnetic Gaussian measure on $\mathbb{Z}_+$ is studied by means of the coarse graining renormalization group method. The result gives information about a singularity behavior of a continued fraction and a time decay rate of a diffusion (random walk) on $\mathbb{Z}_+$.

1. Introduction: Problem and Results

We regard $\mathbb{R}^{\mathbb{Z}_+}$ as a measurable space with the σ-algebra generated by the cylinder subsets of $\mathbb{R}^{\mathbb{Z}_+}$. Let us introduce the notion of ferromagnetic Gaussian measures on $\mathbb{R}^{\mathbb{Z}_+}$. For bounded positive sequences $J=(J_n)_{n\in\mathbb{Z}_+}$ and $g=(g_n)_{n\in\mathbb{Z}_+}$ satisfying

$$\inf_{n\geqq 0} g_n > 0, \tag{1.1}$$

the pair (J,g) is called a *ferromagnetic pair*. We define, for a ferromagnetic pair (J,g), matrices $H(J)$ and $D(g)$ by putting, for $n,m\in\mathbb{Z}_+$,

$$\begin{aligned} H_{nm}(J) &= 0, \quad |n-m|>2,\\ &= J_{n\wedge m}, \quad |n-m|=1,\\ &= -J_{n-1}-J_n, \quad n=m, \end{aligned} \tag{1.2}$$

and

$$D_{nm}(g)=\delta_{nm}g_n, \tag{1.3}$$

where $n\wedge m=\min(n,m)$ and $J_{-1}=0$. The matrix $D(g)-H(J)$ induces a bounded linear operator on $l^\infty(\mathbb{Z}_+)=\{(\phi_n)_{n\in\mathbb{Z}_+} \mid \sup_{n\in\mathbb{Z}_+}|\phi_n|<\infty\}$ and it has a symmetric positive definite inverse (see Lemma 2.1 and 2.2). Then there exists a unique Gaussian probability measure μ_{Jg} on $\mathbb{R}^{\mathbb{Z}_+}$ with mean 0 and covariance $(D(g)-H(J))^{-1}$. We refer to the probability measure μ_{Jg} as the *ferromagnetic Gaussian measure* characterized by (J,g) and write

$$\langle F(\phi)\rangle(J,g)=\int F(\phi)\mu_{Jg}(d\phi)$$