Commun. Math. Phys. 114, 687–698 (1988)

Communications in
Mathematical
Physics

A Boundary Value Problem for the Two Dimensional Broadwell Model*

Carlo Cercignani[1], Reinhard Illner[2] and Marvin Shinbrot[2]

[1] Politecnico di Milano, I-20133 Milano, Italy
[2] University of Victoria, Victoria, Canada V8W 2Y2

Abstract. It is shown that a certain boundary value problem for the steady two-dimensional Broadwell model on a rectangle has a solution. The boundary conditions specify the ingoing particle densities on each side of the rectangle.

1. Introduction

Very little is known about boundary value problems for the Boltzmann equation, even for steady flows. The linearized equation has been much studied [1, 2, 4–9, 13], and there are some results for nonlinear flows near to equilibrium, but that is all we know of, in general. Recently, in an attempt to make further progress on the problem, we began to study boundary value problems for discrete velocity models of the Boltzmann equation. In [11], we showed that, in one dimension, the boundary value problem associated with discrete velocity problems in a slab has solutions quite generally, although we were unable to prove any kind of uniqueness for the solutions we found. In [12], we extended the results of [11] to discrete velocity flows in a half-line. We obtained the result, expected because of the physical analogy, that the solution at infinity is a Maxwellian.

Naturally, one-dimensional steady problems are problems involving ordinary differential equations. In two dimensions, for discrete velocity flows in a domain, virtually nothing is known. In this paper, we present a non-trivial example of the solution of a boundary value problem associated with a natural 4-velocity model in a rectangle. This is the first example we know of such a result.

The model is easily described. We solve the following problem in the rectangle $R=[0,a]\times[0,b]$:

$$\frac{\partial f^1}{\partial x}+f^1f^2=f^3f^4, \qquad f^1(0,y)=\varphi^1(y), \tag{1.1}$$

* Research supported by the Natural Science and Engineering Research Council Canada under Grants A7847 and A8560