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Group Theoretic Approach to the Open Bosonic String Multi-Loop *S*-Matrix

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Abstract. The new approach to string scattering proposed by the authors is generalized to include multi-loop contributions. As an example, the planar one-loop contribution, including its integration measure, to the open bosonic string *S*-matrix is computed. The external state dependence for any multi-loop contribution is computed and found to be determined by one group theoretic function which is derived.

1. Introduction

In a recent paper [1], hereafter referred to as *I*, a new approach to string theory was given. This method relies on the observation that the remarkable simplicity of string scattering amplitudes is a consequence of duality, overlap conditions and unitarity.

The principal character in this approach is the vertex which depends on the moduli, the actual scattering amplitude being obtained by integrating over the moduli with a suitable measure. By moduli we mean the external moduli, i.e., Koba-Nielsen co-ordinates [2] and the parameters associated with loops. As we shall see these arise in connection with the duality properties of the vertex. In a Feynman graph type of approach [3], which the method discussed here is not, the latter parameters arise from the usual parametric form of the propagator used in string theory [4]. While in the sum over Riemann surface approach [5], they arise as the Teichmüller parameters [6, 7].

The duality property well known in string theory states that one can permute the legs of an amplitude and the result is the same provided that one maintains their cyclic order [8] at least for the case of the open string. In the approach advocated here, the vertex can have its dependence on the external moduli cycled by the application of appropriate conformal transformations on all its legs. There also exists, for each loop, a cyclic transformation which leaves the vertex inert and

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