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Analyticity Properties of Eigenfunctions and Scattering Matrix*

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Abstract. For potentials $V=V(x)=O(|x|^{-2-\varepsilon})$ for $|x|\to\infty$, $x\in\mathbb{R}^3$, we prove that if the S-matrix of $(-\Delta, -\Delta+V)$ has an analytic extension $\tilde{S}(z)$ to a region $\mathcal{O}$ in the lower half-plane, then the family of generalized eigenfunctions of $-\Delta+V$ has an analytic extension $\tilde{\phi}(k,\omega,x)$ to $\mathcal{O}$ such that $|\tilde{\phi}(k,\omega,x)|<Ce^{b|x|}$ for $|\mathrm{Im}\,k|<b$. Consequently, the resolvent $(-\Delta+V-z^2)^{-1}$ has an analytic continuation from $\mathbb{C}^+$ to $\{k\in\mathcal{O}\,|\,|\mathrm{Im}\,k|<b\}$ as an operator $\tilde{R}(z)$ from $\mathscr{H}_b=\{f=e^{-b|x|}g\,|\,g\in L_2(\mathbb{R}^3)\}$ to $\mathscr{H}_{-b}$. Based on this, we define for potentials $W=o(e^{-2b|x|})$ resonances of $(-\Delta+V, -\Delta+V+W)$ as poles of $(1+W\tilde{R}(z))^{-1}$ and identify these resonances with poles of the analytically continued S-matrix of $(-\Delta+V, -\Delta+V+W)$.

Introduction

Analytic continuation of the scattering matrix of a two-body Schrödinger operator $-\Delta+V$ has been established for various classes of the potential V, including exponentially decaying [3] and dilation-analytic, short-range [4] potentials.

Two methods were developed to obtain a unified approach to these two classes of potentials, one [5] based on local spectral deformation techniques in momentum space, the other [9] based on an analytic family of deformations of the underlying momentum-space. These methods cover potentials of the form $V+W$, where $V=O(r^{-2-\varepsilon})$ is radial, dilation-analytic and W is exponentially decaying.

For radial potentials a different method was introduced [6]. The basic idea was that if the resolvent $(-\Delta+V-k^2)^{-1}$ can be shown to have an analytic continuation to a domain $\mathcal{O}$ in the lower half-plane as an operator from a space of exponentially decaying functions to its dual, then $-\Delta+V$ can play the role of $-\Delta$ as background for an exponentially decaying perturbation W, using analytic

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