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A Priori Estimates for $N=2$ Wess-Zumino Models on a Cylinder*

Arthur Jaffe and Andrzej Lesniewski
Harvard University, Cambridge, MA 02138, USA

Abstract. We establish bounds uniform in the ultraviolet cutoff (i.e., in the number of degrees of freedom) for a family of two-dimensional Wess-Zumino models. These estimates are useful in proving existence of the models, as well as in investigating their properties. For example, we require these estimates for the analysis of the supercharge and of the Hamiltonian. These are the fundamental a priori estimates for elliptic regularity in infinite dimensions.

I. Introduction

In this paper we establish the fundamental, elliptic a priori estimates required for our analysis of two-dimensional Wess-Zumino models on a cylinder [1, 2]. These estimates are required for the construction of the models, as well as for the study of their detailed properties. We study the $N=2$ models in this paper as defined in [1, 2]. We follow the notation introduced in [1, 2]. These models are defined on the loop space of functions φ: $T^1 \rightarrow \mathbb{C}$.

The estimates here provide the first steps toward developing an analytic theory of Dirac operators on infinite dimensional manifolds. The extensions of these estimates to the $N=1$ and other frameworks, as well as to more general target spaces, are interesting questions under investigation.

We use the Feynman-Kac representations of [1, 2]. Our estimates generalize the methods used in the construction of the Y_2 and $P(\varphi)_2$ field theory models [3, 4]. The work here reduced the analysis of the models we study to standard estimates developed in Chap. 8 of [3]. Thus constructive field theory provides a suitable framework for this set of problems in infinite dimensional analysis.

It is useful to estimate operator norms of the heat kernel $\exp(-\beta H)$ using Schatten class norms $\|\cdot\|_p$ defined by the l_p summability of the characteristic values. Thus if λ_i are the eigenvalues of $(T^*T)^{1/2}$, the I_p norm is

$$\|T\|_{I_p} = \|T\|_p = \left(\sum_i \lambda_i^p\right)^{1/p} = (\mathrm{Tr}(T^*T)^{p/2})^{1/p}.$$

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