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On the Uniqueness of the Infinite Cluster in the Percolation Model

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Abstract. We simplify the recent proof by Aizenman, Kesten and Newman of the uniqueness of the infinite open cluster in the percolation model. Our new proof is more suitable for generalization in the direction of percolation-type processes with dependent site variables.

1. Introduction

It has long been conjectured that the infinite open cluster of the percolation model is unique (almost surely) whenever it exists. This conjecture was verified affirmatively in the recent paper of Aizenman, Kesten and Newman [1], which is couched in the context of (possibly long-range) percolation on any lattice $\mathscr{L}$. Their proof utilizes several distinct ideas and techniques, some of which have their origins in statistical mechanics. Furthermore, the proof has certain consequences for the "thermodynamic functions" of percolation theory, such as the number of clusters per site and the connectivity functions. On the other hand, there are certain miraculous aspects to the method of proof in [1], and it was in attempting to understand this proof that the ideas of this note evolved. In this note, we present a proof of the uniqueness of the infinite open cluster which uses essentially only one of the main ingredients of [1], namely a large-deviation estimate for a certain random variable defined on large but finite open clusters.

The principal motivation for this work was to understand how one may prove the uniqueness theorem for more general processes than "Bernoulli" percolation. Such a generalization to a class of Gibbs measures will appear in [3]. In addition, we hope that our argument may be useful in approaching the question of the uniqueness of the "incipient infinite cluster" of the percolation process; see [2] and [8]. In related work, Gandolfi, Keane and Russo [4] have shown that the infinite cluster is unique for a certain class of two-dimensional models; their techniques are similar to those of Harris [6] for Bernoulli percolation, and are quite different from the general arguments of [1] and the present paper.

We refer the reader to [1] and [4] for motivation and background.