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The Equilibrium States of the Spin-Boson Model

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Abstract. The temperature states of the spin-boson model consisting of a two-level atom in a Bose field are studied. It is proved that for all temperatures there exists a unique solution, hence there is no spontaneous reflection symmetry breaking.

1. Introduction

Spin-boson models are very popular in solid state physics, quantum chemistry as well as in quantum tunneling. A fairly good introduction to the physics can be found in [1].

Here we are particularly interested in the following model:

$$H = \int dk\, \varepsilon(k) a_k^+ a_k + \sigma_3 \int dk\, \lambda(k)(a_k^+ + a_k) + \mu\sigma_1$$

describing a two-level atom in a boson field. On the basis of physical arguments one assumes that the following conditions are satisfied:

$$\int dk\, \lambda(k)^2 < \infty, \quad \int dk \frac{\lambda(k)^2}{\varepsilon(k)} < \infty; \quad \varepsilon(k) \simeq |k|.$$

In this work we study the temperature states of this model in a rigorous way. The ground state problem will be kept for a future occasion. The main aspect we search for is whether or not there is spontaneous symmetry breaking of the reflection symmetry: $\sigma_3 \to -\sigma_3$, $a_k \to -a_k$. Usual techniques for proving the absence of symmetry breaking are not applicable because the group is finite.

As far as we know there exist only a few rigorous results for this model. In [2] one discusses some results on the spectrum of the proposed Hamiltonian, in [3] a thorough analysis is made of a finite mode approximation of the Hamiltonian. In particular the Hartree–Fock solutions are found to show breaking of the symmetry under the condition $\mu < 2\int(\lambda(k)^2/\varepsilon(k))dk$. In [4] the ground state of the model is analyzed. By functional integration techniques it is shown that no

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