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Rarefactions and Large Time Behavior for Parabolic Equations and Monotone Schemes*

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Abstract. We consider the large time behavior of monotone semigroups associated with degenerate parabolic equations and monotone difference schemes. For an appropriate class of initial data the solution is shown to converge to rarefaction waves at a determined asymptotic rate.

1. Introduction

Our main point of interest is the large time behavior of two solution operators, one continuous, the other discrete, when acting on a certain class of initial data.

The continuous example is the solution to the class of degenerate parabolic equations of the type

$$u_t + f(u)_x = A(u)_{xx}, \tag{1.1}$$

where u is scalar, f is convex and $A'(u) \geqq 0$. When $A(u) = |u|^\gamma \cdot u, \gamma > 0$, we have the convective porous medium equation.

The discrete example is the class of monotone difference schemes for the scalar conservation law ((1.1) with $A \equiv 0$). We write the scheme in the following way:

$$u^{n+1}(x) = u^n(x) - \lambda \Delta_d(g(u^n(x - p_0 d), \ldots, u^n(x + q_0 d)), \tag{1.2}$$

where we chose $x \in \mathbf{R}$ rather than on a mesh

$$\lambda = \frac{\Delta x}{\Delta t}, \quad (\Delta_d u)(x) = u(x) - u(x - d), \quad p_0 \geqq 0, \quad q_0 > 0, \quad d > 0,$$

and several conditions on the numerical flux g will be specified. The parameter d is not necessarily small.

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