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Relative Index Theorems and Supersymmetric Scattering Theory

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Abstract. We discuss supersymmetric scattering theory and employ Krein's theory of spectral shift functions to investigate supersymmetric scattering systems. This is the basis for the derivation of relative index theorems on some classes of open manifolds. As an example we discuss the de Rham complex for obstacles in $\mathbb{R}^N$ and asymptotically flat manifolds. It is shown that the absolute or relative Euler characteristic of an obstacle in $\mathbb{R}^N$ may be obtained from scattering data for the Laplace operator on forms with absolute or relative boundary conditions respectively. In the case of asymptotically flat manifolds we obtain the Chern-Gauss-Bonnet theorem for the L^2-Euler characteristic.

1. Introduction

Supersymmetry is a recent concept of quantum field theory which has also interesting applications in mathematics. Recall that a supersymmetry theory consists of a Hilbert space $\mathscr{H}$ together with a unitary involution τ in $\mathscr{H}$ [in physics τ is usually denoted by $(-1)^F$] and selfadjoint operators $Q_i, i=1,\dots,N$, acting in $\mathscr{H}$ and satisfying

$$\tau Q_i + Q_i \tau = 0, \quad i=1,\dots,N,$$

$$Q_i Q_j + Q_j Q_i = 0, \quad i \neq j.$$

Moreover, the Hamiltonian H of the theory satisfies

$$H = Q_i^2$$

for each i. Q_i is called supercharge and H a Hamiltonian with supersymmetry.

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