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Spectral Curves and the ADHM Method

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Abstract. For a general monopole the algebraic curves defined by Nahm are shown to be the same as the spectral curves.

1. Introduction

It is shown in [6] that an $SU(n)$ monopole or solution of the Bogomolny equations with maximal symmetry breaking at infinity has associated to it a collection of $n-1$ algebraic curves $S_1, \ldots, S_{n-1}$ whose intersections $S_i \cap S_{i+1}$ decompose as $S_{i+1,i} + S_{i-1,i}$. Following [3] these are called *spectral curves* and a general monopole is completely determined by these curves and the splitting of $S_i \cap S_{i+1}$ [6].

Nahm, using his adaption of the Atiyah–Drinfeld–Hitchin–Manin (ADHM) approach to instantons, has shown how to associate to a monopole a possibly different set of $n-1$ curves. The purpose of this paper is to show that the spectral curves and the Nahm curves always coincide for a general monopole. It provides in the particular case of $SU(2)$ a replacement for [4] Sect. 7, which is incorrect due to a sign error.

In Sect. 2 some basic facts and the construction of Nahm's spectral curve is reviewed. Each of Nahm's curves is constructed from a vector space W_z and three endomorphisms $T_i(z) \in \mathrm{End}(W_z)$, $i = 1, 2, 3$.

Section 3 shows how to realize the spectral curves by applying Nahm's methods to a different vector space V_z and endomorphisms $H_i(z) \in \mathrm{End}(V_z)$. In the final section an isomorphism is constructed from V_z to W_z which intertwines $H_i(z)$ and $T_i(z)$ for $i = 1, 2, 3$, and thereby proves the identity of the curves. This isomorphism is provided by the Penrose correspondence between solutions to zero rest mass field equations and elements of sheaf cohomology groups, reduced to the three-dimensional situation. It is the link between the algebraic geometry of the spectral curve and the analytical origin of the Nahm curve.

As this paper relies heavily on the methods of [4] we shall, in the interests of brevity, assume that the reader has a copy of it near at hand. Throughout we shall adopt the same notation for a holomorphic bundle and its sheaf of sections, in particular $\mathcal{O}(k)$ will denote the holomorphic line bundle one $\mathbf{P}_1$ of Chern class k or its pullback to $T\mathbf{P}_1$.