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The Poisson Algebra of the Invariant Charges of the Nambu-Goto Theory: Casimir Elements

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Abstract. The reparametrization invariant "non-local" conserved charges of the Nambu-Goto theory form an algebra under Poisson bracket operation. The center of the formal closure of this algebra is determined. The relation of the central elements to the constraints of the Nambu-Goto theory is clarified.

I. Introduction

By now it is well known that the classical Nambu-Goto theory of bosonic closed strings moving in $d \geqq 3$ space-time dimensions possesses infinitely many independent, reparametrization invariant, "non-local" conserved charges $\mathscr{Z}^{\pm(K)}_{\mu_1 \ldots \mu_N}$ which serve as infinitesimal generators of active symmetry transformations [1, 2] and which can be carried over to the quantum theory at least in the WKB-approximation [3]. The Poisson algebra $\mathfrak{h}^{\pm}_{\mathscr{P}}$ (for given numerical values of the linear momenta $\mathscr{P}_\mu$) of these "invariant" charges $\mathscr{Z}^{\pm(K)}_{\mu_1 \ldots \mu_N}$ has been analyzed in some detail [4]. A maximal abelian subalgebra has been identified for the case $\mathscr{P}^2 = \mathfrak{m}^2 > 0$ and it has been established that there are no Casimir elements in the algebra $\mathfrak{h}^{\pm}_{\mathscr{P}}$ itself [5].

In the present paper I shall determine the Casimir elements contained in the formal closure $\overline{\mathfrak{h}^{\pm}_{\mathscr{P}}}$ of $\mathfrak{h}^{\pm}_{\mathscr{P}}$ for $\mathscr{P}^2 = \mathfrak{m}^2 > 0$ and point out their relation to the constraints of the Nambu-Goto theory. As it turns out, there are exactly two such independent Casimir elements: one in $\overline{\mathfrak{h}^{+}_{\mathscr{P}}}$ and one in $\overline{\mathfrak{h}^{-}_{\mathscr{P}}}$. The corresponding formal series in the Fourier components $A^{\pm m}_\mu(\tau)$ of the string variables $u^{\pm}_\mu(\tau, \sigma) = p_\mu(\tau, \sigma) \pm M^2 x'_\mu(\tau, \sigma)$, $0 \leqq \sigma \leqq 2\pi$, $u^{\pm}_\mu(\tau, \sigma + 2\pi) = u^{\pm}_\mu(\tau, \sigma)$, $\mu = 0, 1, \ldots, d-1$, where τ is a time-like and σ is a space-like parameter, can be derived from the integrals

$$\oint_{\tau = \text{const}} d\sigma [(u^{+\mu}(\tau, \sigma) u^{+}_\mu(\tau, \sigma))^2]^{1/4} \quad \text{and} \quad \oint_{\tau = \text{const}} d\sigma [(u^{-\mu}(\tau, \sigma) u^{-}_\mu(\tau, \sigma))^2]^{1/4},$$

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