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Extension of Pirogov–Sinai Theory of Phase Transitions to Infinite Range Interactions. II. Phase Diagram

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Abstract. This paper is the second part of our attempt of an extension of the Pirogov–Sinai theory of phase transitions at low temperatures, applicable to the lattice spin systems with finite range interactions, to the systems with infinite range interactions. Employing the cluster expansion method developed in Part I and modifying the notion of the truncated contour model introduced by Zahradnik, we extend the Pirogov–Sinai result on the structure of phase diagrams to our situations. As an application, we apply our result to Potts models with infinite range interactions.

1. Introduction: Main Result

We continue our attempt to extend the Pirogov–Sinai (PS) theory of phase transitions to classical (discrete and bounded) lattice spin system with *infinite range interactions*. In Part I [3], we have developed a cluster expansion method, and shown that under appropriate assumptions on the interactions the cluster expansion converges for each stable ground state. Thus for each stable ground state we can construct via the cluster expansion method an infinite volume limit pure Gibbs state [3]. In this paper we use the cluster expansion method and some modifications of Zahradnik's version of the PS theory [6] to investigate the structure of the phase diagram for a given system. In order to show how our result can be applied, we study the phase diagrams of Potts models with infinite range interactions.

The PS theory applies to general bounded and discrete lattice spin systems of the following types: The particles (spins) interact with arbitrary finite range periodic potentials. The Hamiltonian of the system, H_0, has n ground states which satisfy the Peierls condition [4, 5]. Consider the structure of the phase diagram of the Hamiltonian

$$H_\lambda = H_0 + \sum_{i=1}^{n-1} \lambda_i H_i$$

in the $n-1$ dimensional parameter space. The H_i are perturbations which lift the

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