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The Invariant Charges of the Nambu-Goto Theory: Their Geometric Origin and Their Completeness

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Abstract. We give an alternative construction of the reparametrization invariant "non-local" conserved charges of the Nambu-Goto theory which elucidates their geometric nature and their completeness property.

I. Introduction

In a series of publications [1–3] the present authors have shown that the classical Nambu-Goto string theory possesses infinitely many independent, reparametrization invariant, "non-local" conserved charges $\mathscr{Z}^{\pm(K)}_{\mu_1\ldots\mu_N}$, which serve as infinitesimal generators of active symmetry transformations. The Poisson algebra of these "invariant charges" $\mathscr{Z}^{\pm(K)}_{\mu_1\ldots\mu_N}$ has been analyzed in great detail [2, 4].

In the present article we shall give a geometric interpretation of the invariant charges of the Nambu-Goto string in its Euclidean version. In particular, we shall address the question to what extent a given choice of values for the invariant charges fixes the "trajectory" surface of the string in d-dimensional Euclidean space $\mathbb{R}^d$.

The geometric interpretation will be formulated in terms of homology and cohomology classes: A given complete minimal ("trajectory") surface Σ can be complexified to a Riemann surface in $\mathbb{C}^d$. This surface can be described by a d-tuple of multi-valued analytic functions $w_\mu = w_\mu(z)$ such that $w'_\mu(z) = \frac{d}{dz} w_\mu(z)$ are single-valued analytic functions in a canonical region Ω satisfying the complex constraint equation $\sum_\mu w'^2_\mu(z) = 0$ for $z \in \Omega$ [5]. The summation extends from $\mu = 1$ to $\mu = d$. The canonical region Ω typically has the form of an annulus $1 < |z| < e^{\lambda_1}$ minus $(n-2)$ concentric circular (two-sided) slits along a connected part of: $|z| = e^{\lambda_i}$, $i = 2, \ldots, n-1$; $\lambda_1 > \lambda_2 \geqq \lambda_3 \geqq \ldots \geqq \lambda_{n-1} > 0$; $n = 2, 3, \ldots, \infty$ [6, 7]. The values of the invariant charges for the given minimal surface are periods of certain analytic differentials – constructed from the d-tuple of analytic functions – along a closed curve γ in the canonical region (corresponding to a closed curve $\mathscr{C}$ on the minimal surface Σ in $\mathbb{R}^d$). As such they depend on the curve γ only via its homology class $[\gamma]$. This is the Euclidean version of the statement that in Minkowski space the charges in question are reparametrization invariant and conserved.