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Scaling Laws in the Vortex Lattice Model of Turbulence*

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Abstract. The lattice vortex model of the inertial range in turbulence theory is reviewed; the model consists of an array of vortex tubes whose axes coincide with the bonds on a regular lattice, subjected to random stretching and successive scaling, and constrained by conservation laws for energy, specific volume, circulation, helicity, and an energy/vorticity relation. The scaling laws for vorticity are examined in detail, a Hausdorff dimension for the "active" portion of the vortex array is calculated, the origin of intermittency is exhibited, and it is pointed out that the Kolmogorov $-5/3$ power law already accounts for intermittency effects.

Introduction

The inertial range in turbulence is the range of scales far enough from the scale of the driving forces to sustain universal statistics yet not so small that viscous effects are important in their dynamics. The analysis of the inertial range is important for the understanding of turbulence and for the design of practical modeling methods. Numerical calculations designed to elucidate the structure of the inertial range, in particular by vortex methods [2, 5], display surprising patterns of complexity and have not been convincingly reconciled with the qualitative theory of Kolmogorov, Kraichnan, and others [9, 11, 14]. These calculations do provide evidence that vortex tubes stretch, bend and bond into fractal structures.

The lattice model we shall examine stands half way between a straightforward vortex calculation and a qualitative model. It was suggested by the calculations in [5], and affords a way of constraining the simple cascade models of the inertial range to obey the basic conservation properties of the Euler equation. Some aspects of the calculations in [5] have been challenged, in particular by Greengard [10], but their usefulness as a qualitative guide is not impaired, except for one issue

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