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Supertori are Algebraic Curves*

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Abstract. Super Riemann surfaces of genus 1, with arbitrary spin structures, are shown to be the sets of zeroes of certain polynomial equations in projective superspace. We conjecture that the same is true for arbitrary genus. Properties of superelliptic functions and super theta functions are discussed. The boundary of the genus 1 super moduli space is determined.

1. Introduction

The application of methods from the theory of Riemann surfaces has lead to great progress in string theory [1–3], as physicists have benefited serendipitously from a century of development of this classical branch of mathematics. The theory of super Riemann surfaces (SRS's) should play a similar foundational role in superstring theory. Here, however, physicists have not found the necessary mathematics already developed, but have had to create the theory themselves along with its applications [4–8]. During the past two years, supersymmetric generalizations have been found for many aspects of Riemann surface theory. Such deep results as the representation of surfaces by Fuchsian groups and the structure of the Teichmüller space have been generalized, while some relatively trivial concepts such as the period matrix have resisted generalization.

A basic property of Riemann surfaces is that they are algebraic curves: any compact Riemann surface can be analytically embedded in a complex projective space as the locus of points whose coordinates satisfy some polynomial equations. This allows the study of Riemann surfaces by the techniques of algebraic geometry and is the key to deep connections between Riemann surfaces and number theory. The algebraic aspect of Riemann surfaces has appeared in string theory in the study of orbifolds [9], and is central to the description of fermions on a Riemann surface via the KP hierarchy of soliton equations [10–13]. Friedan and Shenker

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