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A Block Spin Construction of Ondelettes*
Part II: The QFT Connection

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Abstract. We apply the Lemarié basis of ondelettes to the Battle–Federbush cluster expansion for the ϕ_3^4 quantum field theory. Since there is no infrared problem for this model, we also show how the large-scale ondelettes can be thrown away and replaced by unit-scale functions. Finally, we apply the block spin machine of Part I to the construction of exponentially localized ondelettes orthogonal with respect to the free, massless action of the scalar field.

1. Introduction

Ondelettes [1–5] have an important application to phase cell cluster expansions in quantum field theory [6–11]. They provide a very natural setting for the expansion of any interacting vacuum expectation that can be regarded as a small perturbation of a Gaussian expectation. The minimum-scale and finite-volume cutoffs are simultaneously removed by such an expansion because elementary stability is the only kind of stability needed. Moreover, such a decomposition of phase space has renormalization group ideas built into it.

A cluster expansion was developed in [7] which was based on expansion functions (a certain Bessel potential of certain L^2-ondelettes) with respect to which the free Euclidean boson field ϕ with mass m has a diagonal covariance. Thus the expansion decouples only the interaction when applied to the ϕ_d^{2n} and Y_d models. The specific ondelettes that were used in [7], however, suffered from a serious lack of regularity that affected the positivity of the ϕ_3^4 interaction with respect to the random variables corresponding to such expansion functions. Consequently, the diagonal-covariance expansion was applied to a hierarchical version [8] of the ϕ_3^4 model. Williamson [9] controlled the ϕ_3^4 model by using the L^2-ondelettes directly as expansion functions and decoupling the resulting non-diagonal covariance as well as the interaction. This choice solves the positivity problem for the Battle–Federbush ondelettes.

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