The Algebra Formed by the Invariant Charges of the Nambu-Goto Theory: Identification of a Maximal Abelian Subalgebra

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Abstract. Continuing the analysis of the algebraic structure of the invariant charges of the Nambu-Goto theory, we identify a complete set of commuting observables for the bosonic strings.

I. Introduction

In a recent publication [1] the present authors have studied the general structure of the algebras $\mathfrak{h}_{\mathscr{P}}^{\pm}$ formed by the conserved observable tensor charges of the bosonic Nambu-Goto string theory under tensor multiplication and Poisson bracket operation. The complete infinite dimensional algebra of infinitesimal classical symmetry transformations

$$\mathfrak{g} = \mathrm{so}(1, d-1) \oplus (\mathbb{M}^d \oplus \mathfrak{h}_{\mathscr{P}}^+ \oplus \mathfrak{h}_{\mathscr{P}}^-)$$

turned out to be a minimal extension of the Poincaré algebra involving string degrees of freedom. Both $\mathfrak{h}_{\mathscr{P}}^+$ and $\mathfrak{h}_{\mathscr{P}}^-$ are graded algebras

$$\mathfrak{h}_{\mathscr{P}}^{\pm} = \bigoplus_{\ell=0}^{\infty} V^{(\ell)}(\mathfrak{h}_{\mathscr{P}}^{\pm})$$

such that – with the symbol $\{\cdot, \cdot\}$ standing for the Poisson bracket operation –

$$\{V^{(\ell)}(\mathfrak{h}_{\mathscr{P}}^{\pm}), V^{(\ell')}(\mathfrak{h}_{\mathscr{P}}^{\pm})\} \subset V^{(\ell+\ell')}(\mathfrak{h}_{\mathscr{P}}^{\pm}).$$

In this paper we shall identify a *maximal* abelian subalgebra

$$\mathfrak{a}_{\mathscr{P}}^{\pm} \subset \mathfrak{h}_{\mathscr{P}}^{\pm}$$

provided: $\mathscr{P}^2 = m^2 > 0$, whose elements ultimately should serve to distinguish the states of irreducible positive energy representations of $\mathfrak{g}$.

Since $\mathfrak{h}_{\mathscr{P}}^+$ and $\mathfrak{h}_{\mathscr{P}}^-$ are mutually commuting, isomorphic Poisson bracket algebras, we take the liberty to restrict our attention to $\mathfrak{h}_{\mathscr{P}}^+$, and to suppress the superscript $+$ in the sequel.