Commun. Math. Phys. 113, 649–657 (1988)

Communications in
Mathematical
Physics

Quantization of the Kepler Manifold

Bruno Cordani

Dipartimento di Matematica dell'Università, via Saldini 50, I-20133 Milano, Italy

Abstract. A representation of $SO(2,n+1)$, the maximal finite dimensional dynamical group of the n-dimensional Kepler problem, is obtained by means of (pseudo) differential operators acting on $L_2(S^n)$. This representation is unitary when restricted to $SO(2)\otimes SO(n+1)$, i.e. to the physically relevant subgroup.

1. Introduction

A great number of works have been devoted to the Kepler Problem (KP) but the last word is yet to be said, especially with regard to quantization in the sense of Kostant-Souriau [1–3]. We briefly explain the basic concepts (see e.g. [4–6] and references quoted herein).

The n-dimensional KP, $n \geqq 2$, is the Hamiltonian system on the phase space $T^*(\mathbb{R}^n - \{0\})$ with the Hamiltonian

$$H(q,p) = \frac{1}{2}p^2 - \frac{1}{q}, \tag{1.1}$$

q_k and p_k being canonical coordinates. Owing to the collision orbits the flow is not complete. After regularization (that amounts to compactifying each cotangent space to the configuration manifold by adding the point at infinity) and exchange between coordinates and momenta, the phase space becomes symplectomorphic to the so-called "Kepler manifold," i.e. $T^+S^n := T^*S^n$ – null section. This phase space turns out to be a coadjoint orbit (more exactly: the most singular orbit) of the dynamical group $SO(2,n+1)$. For negative energy the maximal compact subgroup $SO(2)\otimes SO(n+1)$ of the dynamical group is physically relevant: its generators are to be identified respectively with the Hamiltonian and the other constant of motion, i.e. angular momentum and Runge-Lenz-Laplace vector.

This analysis at "classical level" allows us to define in an unambiguous way what we mean for "quantization" of the Kepler manifold: an Almost-Unitary Irreducible Representation (AUIR) of $SO(2,n+1)$ through (pseudo) differential operators acting on L_2 functions on the n-dimensional sphere S^n. We do not