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Double Wells: Perturbation Series Summable to the Eigenvalues and Directly Computable Approximations

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Abstract. We give a rigorous proof of the analyticity of the eigenvalues of the double-well Schrödinger operators and of the associated resonances. We specialize the Rayleigh–Schrödinger perturbation theory to such problems, obtaining an expression for the complex perturbation series uniquely related to the eigenvalues through a summation method. By an approximation we obtain new series expansions directly computable, still summable, which, in the case of the Herbst–Simon model, can be given in an explicit form.

1. Introduction

In many problems of physical interest the Rayleigh–Schrödinger perturbation theory provides divergent power series expansions for the eigenvalues. In several cases not only are such expansions asymptotic to the perturbed eigenvalues, but also the correspondence is one-to-one through summation methods such as the Borel one ([14, 9, 2]). In other circumstances, like the Lo Surdo-Stark effect ([14, 6]), it has been proved that the Borel sum of the series in complex directions of the parameter defines the resonances of the problem.

As for the double-well Schrödinger operators, the situation is not completely satisfactory, since the perturbation series is summable in complex directions of the parameter to the "resonances" ([7, 8]) and thus the question is how to obtain the eigenvalues. This problem of perturbation theory could have connections in other areas, such as constructive field theory.

We believe we provide here the conclusive answer to this kind of problem with this type of techniques. In particular we define complex series expansions which are summable in complex directions of the parameter; the sums can be analytically continued to the real axis and are related to the eigenvalues in a simple way. Such series expansions are directly computable by means of approximations as we shall suggest. Better approximations can be obtained in the framework of this theory also by means of semiclassical methods.

In order to examine in more detail these questions, we restrict ourselves to

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