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Super Riemann Surfaces: Uniformization and Teichmüller Theory

Louis Crane[1] and Jeffrey M. Rabin[2]*

[1] The Institute for Advanced Study, Princeton, NJ 08540, USA

[2] Enrico Fermi Institute and Department of Mathematics, The University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637, USA

Abstract. Teichmüller theory for super Riemann surfaces is rigorously developed using the supermanifold theory of Rogers. In the case of trivial topology in the soul directions, relevant for superstring applications, the following results are proven. The super Teichmüller space is a complex super-orbifold whose body is the ordinary Teichmüller space of the associated Riemann surfaces with spin structure. For genus $g > 1$ it has $3g-3$ complex even and $2g-2$ complex odd dimensions. The super modular group which reduces super Teichmüller space to super moduli space is the ordinary modular group; there are no new discrete modular transformations in the odd directions. The boundary of super Teichmüller space contains not only super Riemann surfaces with pinched bodies, but Rogers supermanifolds having nontrivial topology in the odd dimensions as well. We also prove the uniformization theorem for super Riemann surfaces and discuss their representation by discrete supergroups of Fuchsian and Schottky type and by Beltrami differentials. Finally we present partial results for the more difficult problem of classifying super Riemann surfaces of arbitrary topology.

1. Introduction

Polyakov's bosonic string theory [1] is a theory of maps from a two-dimensional surface Σ into (Euclidean) spacetime, with action

$$S = \int d^2\Sigma \sqrt{g} g^{ab} \partial_a X^\mu \partial_b X_\mu,$$
$$X : \Sigma \to R^{26}. \tag{1.1}$$

The world sheet metric g^{ab} is an auxiliary field which permits the action to be

* Enrico Fermi Fellow. Research supported by the NSF (PHY 83-01221) and DOE (DE-AC02-82-ER-40073). Present address: Department of Mathematics, C-012, University of California at San Diego, La Jolla, CA 92093, USA