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Polynomial Deformations and Cohomology of Calabi-Yau Manifolds

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Abstract. We study the method of polynomial deformations that is used in the physics literature to determine the Hodge numbers of Calabi-Yau manifolds as well as the related Yukawa couplings. We show that the argument generally presented in the literature in support of these computations is seriously misleading, give a correct proof which applies to all the cases we found in the literature, and present examples which show that the method is not universally valid. We present a general analysis which applies to all Calabi-Yau manifolds embedded as complete intersections in products of complex projective spaces, yields sufficient conditions for the validity of the polynomial deformation method, and provides an alternative computation of all the Hodge numbers in many cases in which the polynomial method fails.

1. Introduction

Compact Kähler Ricci-flat (usually called Calabi-Yau) manifolds, of complex dimension 3 ($\mathscr{M}_{\mathrm{CY}}$), were recently proposed [1, 2] for compactification of certain superstring theories [3]. In this way, one can obtain an $N=1$ locally supersymmetric grand-unified model (hereafter "effective model") in 4-dimensional Minkowski space-time, with the gauge group being a subgroup of $E_6 \times E_8$ and the matter superfields coming in a chiral representation in general.

The massless superfields of the effective model can be represented by harmonic exterior forms on the internal $\mathscr{M}_{\mathrm{CY}}$ [1, 2, 4][1] and consequently the couplings in the effective model are related [6] by certain integrals of products of forms over $\mathscr{M}_{\mathrm{CY}}$, and possibly its nontrivial submanifolds (see also [4, 7]). These integrals, in general, depend on the complex structure and the cohomology class of the Kähler form and are often calculable. Moreover, their relative values and, in particular, the identical vanishing of some of them can be deduced by applying the Wigner-Eckart theorem, noting that the superfields (i.e. the corresponding forms on $\mathscr{M}_{\mathrm{CY}}$)

[1] There does, however, exist a class of superfields, invariant under the Yang-Mills gauge group, that escapes this classification [4, 5], but this is irrelevant for the purpose of our present paper