

Integrable Quantum Systems and Classical Lie Algebras

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Abstract. We have obtained six new infinite series of trigonometric solutions to triangle equations (quantum R -matrices) associated with the nonexceptional simple Lie algebras: $sl(N)$, $sp(N)$, $o(N)$. The R -matrices are given in two equivalent representations: in an additive one (as a sum of poles with matrix coefficients) and in a multiplicative one (as a ratio of entire matrix functions). These R -matrices provide an exact integrability of anisotropic generalizations of $sl(N)$, $sp(N)$, $o(N)$ invariant one-dimensional lattice magnetics and two-dimensional periodic Toda lattices associated with the above algebras.

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1. Introduction

In the theory of two-dimensional integrable systems of quantum field theory and statistical physics a specific importance is attached to the special system of