

Convex Hamiltonian Energy Surfaces and Their Periodic Trajectories

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Abstract. In this paper we introduce symplectic invariants for convex Hamiltonian energy surfaces and their periodic trajectories and show that these quantities satisfy several nontrivial relations. In particular we show that they can be used to prove multiplicity results for the number of periodic trajectories.

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