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A Construction of Two-Dimensional Quantum Chromodynamics

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Abstract. We present a sketch of the construction of the functional measure for the $SU(2)$ quantum chromodynamics with one generation of fermions in two-dimensional space-time. The method is based on a detailed analysis of Wilson loops.

Introduction

In this paper we present a sketch of the construction of the functional measure for the simplest non-abelian gauge theory with fermions: the two-dimensional quantum chromodynamics. We shall concentrate on the main technical steps and estimates; for more details the reader is referred to [1].

Our methods are rather limited to the two-dimensional case since we use the solvability of the lattice pure gauge theory. However, the way we incorporate fermions into the theory can be, in principle, repeated in 3-dimensions. This may be important for further research since the interaction with fermions has been much less rigorously studied than with the Higgs fields [2], contrary to one's intuition that non-selfinteracting fermions should be easier to treat than self-interacting bosons. We speculate also on the implementation of the renormalization group ideas which look quite natural in our framework.

The paper consists of three parts. Section 1 contains the description of our methods and some preliminary constructions. Main results with sketches of proofs are given in Sect. 2. Finally in Appendix 1 one can find the necessary facts from measure theory and in Appendix 2 the proof of the basic estimate from lattice gauge theory.

1. From Wilson Loops to Functional Measure

In this section we collect miscellaneous facts about the orbit space, Wilson loops and some lattice gauge theory results and explain the main idea of our approach.

Let $\mathscr{A}$ be the space of gauge potentials (connections) on $\mathbb{R}^2$. These are one forms $A_x dx + A_y dy$ with coefficients taking values in a Lie algebra $\mathfrak{G}$ of a compact