

General Proof of Osterwalder-Schrader Positivity for the Wilson Action

P. Menotti¹ and A. Pelissetto²

¹ Dipartimento di Fisica dell'Università di Pisa, and INFN, Sezione di Pisa, I-56100 Pisa, Italy

² Scuola Normale Superiore, Pisa, and INFN, Sezione di Pisa I-56100 Pisa, Italy

Abstract. We extend the proof by Osterwalder and Seiler of reflection positivity of lattice gauge theories with fermions to the case in which the reflection is performed with respect to planes containing sites. Our proof applies to all observables and relies on the particular structure of the Wilson action. This assures the positivity of the correlation functions $\langle \theta(F)F \rangle$ for any gauge invariant function F and at all euclidean time separations as of interest in Monte Carlo calculations.

1. Introduction

A problem in lattice gauge theories [1] is that of proving the Osterwalder-Schrader [2] positivity condition which is one of the fundamental ingredients for reconstructing Wightman theory from euclidean theory. From the practical viewpoint the positivity condition excludes the existence of ghosts from the physical sector of the theory.

Lüscher [3] and independently Osterwalder and Seiler [4] have addressed the problem in two different ways. Lüscher starting from the Wilson action builds up a Hilbert space as a Fock space derived from equal time fields and explicitly constructs a transfer matrix [5] which he proves to be positive definite. In this context Osterwalder-Schrader positivity is proved for gauge invariant operators constructed from the fields belonging to given time slices. Osterwalder and Seiler on the other hand prove the Osterwalder-Schrader positivity condition directly from the functional integral for all gauge invariant quantities. Their procedure is sufficient to construct a positive hamiltonian on a Hilbert space; however they prove the positivity of $\langle \theta(F)F \rangle$ only by reflecting with respect to a plane cutting the time links in half (i.e. for “odd” separations). This situation is not completely satisfactory for the Monte-Carlo calculations where one computes correlation functions of arbitrary gauge invariant composite operators at all (even and odd) distances.

In this note we shall prove the Osterwalder-Schrader positivity condition for all gauge invariant quantities for any separation in the case of the Wilson action.