Commun. Math. Phys. 113, 353-368 (1987)

Communications in Mathematical Physics

Fermionic Path Integration and Grassmann Brownian Motion*

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Abstract. A Grassmann probability theory, with anticommuting random variables and stochastic processes, is developed using an extension of Berezin integration to infinite dimensional spaces. A Kolmogorov-type consistency condition allows integration on spaces of paths in anticommuting space. One particular stochastic process, Grassmann Brownian motion, is described and the associated measure used to give a path-integral formula for the kernel of the evolution operator in fermionic quantum mechanics. The Fourier mode expansion of Grassmann Brownian motion is derived.

1. Introduction

Path integration techniques are among the most powerful in present-day quantum physics; the quantisation of a new type of theory often proceeds, at least in the first instance, by summing over everything, with more or less plausible weights. While this is a good heuristic procedure, and formal manipulation of path integrals often gives extremely valuable insight, ultimately the true meaning (if any) of the path integrals in a theory should be established. This is particularly true in the case of path integrals for fermions, where the integrals are not even the limits of sums, and standard measure theoretical results do not apply. For instance, it is usually assumed that the sum over paths can be replaced by a sum over fourier modes, which is far from obvious when the word sum is being used in a formal sense. (In fact one result of this paper will be to justify this procedure, and to give the correct normalization.)

The analytic theory of path integration in imaginary time bosonic quantum mechanics is fully understood. The basic approach, (which goes back to ideas of Wiener, Feynman and Kac), together with many applications (and references to original work) is described in the excellent book of Simon [1]. In this paper an analytic theory of fermionic path integration is constructed, in a manner as closely

* Research supported by the Science and Engineering Research Council of Great Britain under advanced research fellowship number B/AF/687